Scheduling Counts in the Instantaneous and Progressive Count Methods for Estimating Sportfishing Effort

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Abstract.—There are two methods for estimating sportfishing effort that are based on instantaneous count survey methods. Prog, a complete count of boats or anglers on a specified body of water made at a randomly selected instant of time in a given time interval, provides an unbiased estimate of the average number of boats or anglers fishing in that interval. Consequently, the product of the number counted times the duration of the interval provides an unbiased estimate of the fishing effort during the interval. It often happens that this method is not practical because the entire body of water cannot be viewed in an instant. That is, the survey agent might have to travel for a considerable period of time in order to view all locations on the body of water. A second method for estimating effort involves having a survey agent travel along a defined route that covers the entire fishing area, and having the agent count all anglers encountered throughout the day. Although this method is sometimes referred to as an instantaneous count method because each location is viewed for an instant, we prefer to refer to methods in which counts of anglers take an appreciable amount of time as progressive count methods. We discuss a variation of the progressive count approach in which anglers encountered are counted while the survey agent makes a single circuit through the fishing area, timing the time required to make the circuit and count the anglers encountered. The approach is analogous to the method used for estimating effort in the North American Sportfish Survey. Because the likelihood of encountering an angler is not constant throughout the day, the time required to complete the survey varies. A correction factor is required to account for any changes in the likelihood of encountering anglers. The correction factor can be determined by a variety of methods and is a function of the length of the survey area. The correction factor is used to scale the effort estimate to the duration of the survey time interval. Consequently, the product of the number counted by the survey agent and the duration of the survey time interval provides an unbiased estimate of the fishing effort during the interval. An estimate of the total catch is obtained by multiplying the number of anglers counted by the duration of the survey time interval. The estimate of the total catch is obtained by multiplying the estimated effort by an estimate of the catch rate. The origin of this method is rather obscure and was reviewed by Lambour (1961).

Letting the number counted be denoted by C and the duration of the time interval by T, we have

\[
\hat{E} = CT. \tag{1}
\]

where \(\hat{E}\) is the estimated effort. If \(C\) is the number of
of boats counted and \( T \) is expressed in hours, then \( f_i \) estimates the boat-hours of fishing. An obvious generalization of this method would be to make several (independent, randomly) scheduled counts during the time interval and use the arithmetic mean of the numbers counted, \( C \). Thus, the estimated effort would be

\[
\bar{f} = C/T.
\]

Alternatively, a systematic sample of counts could be obtained, say every hour, with the timing of the first count (within the first hour) selected randomly (see Cochran 1977 for a discussion of systematic random sampling). These variations on the estimator in equation (1) also provide unbiased estimates.

Robson (1961) described a similar method in which the survey agent counts anglers all day long while traveling at constant speed through the fishing survey area. The agent thus makes \( C \) circuits (\( C > 0 \)) through the survey area during the day. The entire survey area is observed during a circuit. Each circuit through the area yields a count of anglers, where a count now refers to the number of anglers (or boats) observed by the agent while making the circuit. Note that although the time required to make a count (i.e., travel one circuit) is no longer an "instant," the survey agent is continuously moving and the count is made in such a way that each location in the survey area is observed for only an instant. For this reason, this method is sometimes referred to as an instantaneous count method. There are three requirements for proper use of the method: (1) the starting location along the survey agent's route is chosen randomly, (2) the direction of travel is chosen randomly (from the two alternatives), and (3) the survey agent's speed of travel is greater than that of all the anglers while the anglers are fishing (but not necessarily when the anglers are traveling from one location to another).

A special case of the method described by Robson (1961) is the case in which the survey agent makes one circuit that takes all day. In this case, the expected number of anglers counted is equal to the fishing effort in angler-days (i.e., in 8-h-long angler-days). Expressing fishing effort in 8-h angler-days is rather awkward, and in general one would probably wish to multiply the number of anglers counted during the day by the number of hours in the survey agent's day to obtain an unbiased estimate of the number of angler-hours of fishing effort occurring during the agent's workday.

The terminology for creel survey methods has not been standardized and can be confusing. For the purposes of this paper, we will refer to methods in which counts are made in a brief period of time as instantaneous count methods, and following Lambrou (1961), we will refer to methods in which counts are made over an appreciable period of time as progressive count methods.

Unfortunately, there is widespread misunderstanding of the ways these methods work. Four common errors we have noted are:

1. Interpreting the instantaneous or progressive count as an estimate of the number of trips in the day rather than as an estimate of number of angler-days of effort;
2. Failing to randomize the count time for the instantaneous count method (e.g., always making the count at the midpoint of the day);
3. Having the survey agent in progressive count schemes interrupt the counting for appreciable amounts of time in order to interview anglers and thus not traveling at a constant speed; and
4. Improperly scheduling the count for a modification of the progressive count method in which a single count is made during the day and the time required to make the count is appreciable but less than a full day.

Use of the instantaneous count as an estimator of the number of trips in the day (error 1) results in a negative bias that can be severe. If the count is multiplied by an estimate of the average trip duration (obtained, say, from interviews at access points) then this would (in general) be an un-biased estimate of the number of angler-hours of fishing effort. Little can be said about the consequence of failing to randomize the count time in the instantaneous count method (error 2) other than to point out that the resulting bias is unpredictable and can be very large depending on the daily pattern of fishing effort. The progressive count problem described as error (3) is one in which the survey agent stops repeatedly to interview anglers, can be shown to cause a negative bias that may be serious. Wade et al. (1991) showed, however, that if several checkpoints are established along the survey agent's path and the agent is forced to arrive at the checkpoints at specified times by speeding up if necessary (thus bringing the agent back on schedule several times a day), then the bias can be made negligible. Robson's (1991) did not allow for agent travel speed to vary with the density of anglers (anglers pr
One usually wants to estimate the fishing effort during the entire day rather than during a 1.5-h portion of the day. Suppose the workday is 6 h long (T), and the sampled 1.5-h block of time is selected "randomly" from the six hours (see below). Then, the fishing effort estimated for the particular block of time (by equation 2) is an unbiased estimate of the mean fishing effort in all such 1.5-h blocks. As there are 4 blocks (s) of duration 1.5 h in a 6-h workday, an unbiased estimate of the total fishing effort in the 6-day work is given by

\[ \bar{E} = \frac{C}{s} = \frac{C}{1.5} \times 4. \]

It remains only to specify methods for picking the starting time. A frequently used, but erroneous, procedure is to select the starting time randomly with continuous uniform probability in the interval (0, T). For example, if the count takes 1.5 h (s), and the study period is 6 h (T) in duration, the starting time may be scheduled randomly in the interval from time 0 to time 4.5 h. (The starting time cannot be after 4.5 h, because the survey agent will not finish before the end of the workday.) Unfortunately, this procedure does not result in all times having equal probability of being sampled (Figure 1). For example, the only way to be counting at time 0 is to select the starting time to be 0. However, time \( T = 1 \) min will be sampled if time 0 = or time 1 min is selected as the starting time (assuming start times are rounded to the nearest minute). Time 2 = min will be sampled if the start time is 0, 1, or 2, and so on. Thus, this method will provide estimates that are biased towards whatever fishing effort occurs in the middle of the workday.

One way to avoid this problem is as follows. Suppose the duration of the study period, \( T < \) an integer multiple of the time, \( r < \) required to make a count, i.e. \( s < \) now an integer. In the example above, with \( T = 6 \) h and \( r = 1.5 \) h, \( s < 4 \). Then, one simply selects an integer \( T < \) randomly with uniform probability in the closed interval \([0, s]< \) and schedules the count to begin at time \( t < \). Thus, we would select one of the following starting times with equal probability: 0, 1.5 h, 3 h, 4.5 h. This guarantees all times have equal probability of being sampled. However, in order that all combinations of time with geographic location have equal probability, it is necessary to select the starting location for the count randomly. Also, the direction of travel must be randomized. A minor problem with this procedure is that the length of the study period, \( T < \), may not happen to be an

\[ E = \frac{C}{s} = \frac{C}{1.5} \times 4. \]

...
insignificant multiple of the count time \( r \). However, the time required to make a count can usually be adjusted to meet this requirement.

A second way to schedule the count is to pick a start time (in advance) in the interval \((0,7)\) with continuous uniform probability and, if necessary, "wrap" the count around to the beginning of the study period. For example, suppose the start time of a 1.5-h count is scheduled for 5.5 h into a 6-h study period. Then half an hour of the count would be made from time \( = 5.5 \text{ h} \) to time \( = 6 \text{ h} \), and the other hour would be made from time \( = 0 \text{ h} \) to time \( = 1 \text{ h} \). In this case, it is not necessary to randomize the starting location (though this does not hurt) provided the starting location is picked objectively (i.e., without referring to conditions in or affecting the fishery). The direction of travel should still be randomized.

**Example 1—Effect of Incorrect Scheduling**

It is often observed that recreational fishing effort shows a period over time during a day, with few anglers fishing early in the morning or late in the day and most activity occurring in the middle of the day (Parker 1956; McNelis and Triar 1991). This might occur, for example, in a "punch-in" fishery for sunfish and crappie. Let us assume that fishing effort in the first half of the day is the function of time of day depicted in Figure 2. The total fishing effort in the 6-h workday is the area under the curve and is easily seen to be

\[
\text{effort} = \text{area of small rectangle } + \text{area of large rectangle } = (2 \times 3) + (4 \times 5) = 6 + 200 = 206 \text{ angler-hours.}
\]

Suppose it takes 2 h to make a count and a single
count is made each day. We first consider a correct procedure for estimating the fishing effort. Let the starting time for the count be chosen with equal probability from the following list of potential starting times: 0 h, 2 h, 4 h. The starting location along the route is randomly selected with each point along the route having equal probability of being selected, and the direction of travel (e.g., clockwise or counterclockwise along the route) is also selected randomly. Then, as shown in Appendix 3, the number of anglers counted is an unbiased estimate of the average number of anglers fishing in the 2-h period when the count was made. Thus, the number of anglers counted times the duration of the counting activity (i.e., 2 h) is an unbiased estimate of the fishing effort in angler-hours during the 2-h interval. Because the 2-h block was chosen with equal probability from the list of three possible starting times, the estimated effort in the 2-h block of time when the count was made is an unbiased estimate of the average effort in the three 2-h periods in the 6-h workday. Consequently, an unbiased estimate of the effort during the 6-h workday is three times the estimated effort during the period the count was made.

More specifically, if time $= 0$ is chosen as the starting time, the expected value of the count is 3. Thus, the expected value of the estimated effort in the 2-h period when the count was made is $2 \times 3 = 6$, and the expected value of the estimated effort in the 6-h day (given the starting time) is $2 \times 3 = 18$. Similarly, if the starting time is time $= 2$, the expected value of the number counted is 50, and the expected value of the estimated effort for the 6-h period is $2 \times 50 = 3000$; the same holds if the starting time is time $= 4$. Because the three starting times are chosen with equal probability, the expected value of the estimated 6-h effort (averaged over possible starting times) is simply the average of 18, 300, and 300 (i.e., 206 hours).

Now consider that the starting time is incorrect) selected by picking a time with uniform probability in the interval (0 h, 4 h). Then the expected value of the estimated effort obtained from equation (3) is

$$E[C^r] = E[Z] = E[Z][E[C^r]],$$

where $E[Z]$ is in this case and $E[Z]$ and $E[Z]$ represent expectations over stages 1 and 2. Here, the expectation at the second stage refers to the average over all possible starting locations, and the expectation over the first stage refers to the average over all possible starting times. The expectation at the second stage is the expectation of equation (2), which is equal to the fishing effort in the particular 2-h block of time sampled (Robson 1961). Thus,

$$E[C^r] = E[Z],$$

where $Z$ is the fishing effort in the 2-h block of time starting at time $t$, and $E[Z]$ is the average or expectation over all starting times.

The fishing effort for any 2-h block of time is shown in Figure 3. Clearly, in the 2-h block of time starting at $t = 0$, there are 3 anglers $\times 2 = 6$ angler-hours of fishing effort (see Figure 2). Also, in any 2-h block starting in the interval from $t = 2$ to $t = 4$, there are 50 $\times 2 = 100$ angler-hours of effort. Between $t = 0$ and $t = 2$, the amount of fishing effort (in the block of time starting at time...
i) increases linearly with \( i \), as shown in Figure 3. For any starting time \( t \) between 0 and 2, there are 3 anglers fishing for a period of time equal to 2 – \( t \), and there are 50 anglers fishing for 2 time units. Note that

\[
3(2 - t) + 50t = 6 - 3t + 50t = 6 + 47t.
\]

Thus, \( f_i \) is given by

\[
f_i = \frac{6 + 47t}{100}, \quad 0 < t < 2, \quad 2 \leq t < 4.
\]

If the starting time \( t \) is chosen randomly (with uniform probability) from the interval (0, 4), then the expected value of the estimated fishing effort in the randomly selected 2-h block is obtained by integrating \( f_i \) over the range of possible starting times. Thus,

\[
E(F) = \int_0^2 f_i \cdot \frac{6 + 47t}{100} dt = \int_2^4 f_i \cdot \frac{6 + 47t}{100} dt = \frac{1}{4} (6 + 47t)^{2,5} + \frac{1}{4} (100)^{2,5}.
\]

where \( f_i \) is the probability density function for a random variable distributed uniformly over the interval (0, 4). From the above, \( E(F) \) can be seen to be 76.5. This is the expected value of the estimated fishing effort in a randomly selected 2-h block; consequently, by equation (3) the estimate for the 6-h workday would be obtained by multiplying by \( i = 3 \). Thus, the expected value of the estimator would be 239.5 angler-hours, or 11.4% too large when the starting time is chosen incorrectly.

Scheduling Instantaneous Count Times with Non-uniform Probabilities

In this section we present a new generalization to the theory of creel surveys. Let us suppose the fishing day is divided into a large number, \( J \), of time intervals of short duration \( \Delta \). For example, a 6-h fishing day may be divided into 360 intervals of 1-min duration. Suppose, further, that anglers enter and leave the fishery only at the beginning of the time intervals. If the time intervals are sufficiently short that this assumption does not present any difficulties.

Let us suppose that a complete count of anglers can be made in less than a minute and, because anglers do not enter or leave the fishery during a minute time interval, the count is effectively instantaneous. The usual procedure would be to randomly pick one minute, with equal probability assigned to each minute, in order to make the count. That is, each minute has probability \( 1/J \) of being selected.

Suppose, instead, each minute is assigned a probability \( p_i \) of being selected for the count so that

\[
\sum p_i = 1.
\]

Let \( C_i \) be the number of anglers fishing during minute \( i \), and let the estimate of fishing effort during the day be given by the Horvitz-Thompson estimator (see Cochran 1977)

\[
\hat{F} = \sum C_i p_i / \bar{p}
\]

(4)

It is well known that the Horvitz-Thompson estimator is unbiased. Its expected value is, by definition, the sum over all \( J \) possible outcomes of the outcome times the probability of obtaining the outcome. Thus,

\[
E[F] = \sum E[C_i p_i / \bar{p}] / \bar{p}
\]

\[
= \sum C_i / \bar{p} \Delta
\]

\[
= \text{total effort / effort in each minute}
\]

where \( E[\cdot] \) is the expectation operator. This is proof that the estimator is unbiased. The estimator can also be shown to be unbiased when the fishing effort in the \( j \)th interval is estimated by (an unbiased estimator) rather than observed.

The logic of the Horvitz-Thompson estimator can be described as follows. Suppose one believes that 1% of the daily fishing effort occurs during some interval \( i \). Then this implies that one believes that the total effort during the day is equal to that observed in interval \( i \) divided by 0.01. Of course, the fishing effort during minute \( i \) may not be 1% of the total. However, as shown above, the estimator is unbiased regardless of whether the sampling probabilities are closely related to the amount of fishing effort in the corresponding intervals. Whereas the estimator is unbiased regardless of the sampling probabilities, its variance is highly dependent on the choice of probabilities. (If one is correct that 1% of the fishing effort occurs during the minute selected then the estimated fishing effort will be exactly correct; if the actual effort differs greatly from what was anticipated the estimated effort will be far from the actual effort.)

When the pattern of fishing effort over the course of a day is well known, and the sampling prob-
minutes is assigned for the count so

angles fishing during that fishing effort. Horvitz-Thompson

\[ \text{Horvitz-Thompson expected value is, by definition, the expected total of all possible outcomes of the sampling process.} \]

The Horvitz-Thompson estimator involves assigning a sampling probability for every sampling unit (e.g., minute) in the population. There is no reason why we can't replace the set of sampling probabilities with a continuous sampling probability density function and treat time as a continuous, rather than discrete, quantity. Thus, the set of \( P_i \) is replaced with a positive continuous function of time \( P(t), t \in (0, T) \), with

\[ \int_0^T P(t) \, dt = 1. \]

Then the estimator in equation (4) is replaced with the continuous analog

\[ \hat{f}_{HTC} = \frac{C_t}{P(t)} \]

where \( C_t \) is the number of anglers fishing at time \( t \). By arguments analogous to those used for the Horvitz-Thompson estimator, this estimator can be shown to be unbiased.

Example 2—Scheduling Counts with Nonuniform Probabilities

This example is intended to illustrate how the Horvitz-Thompson method is used and is not intended to be realistic. Suppose the relative fishing effort in each minute of a 360-min workday is believed to be as in Figure 4. The area under the curve is \((10 \times 60) + (40 \times 300) = 12,600\) angler-minutes. The portion of the daily total effort believed to occur in minute 1 is \(10/12,600 = 0.0007936\). The same holds true for minutes 2 through 60. For any minute from 61 to 360, the anticipated proportion of the total effort is \(40/12,600 = 0.0031746\). Each minute is assigned a sampling probability equal to the anticipated proportion of the total effort occurring in the minute (Table 1). The estimated total effort is computed as follows. If the minute sampled is in the interval [1, 60], then the estimated effort equals the number counted divided by 0.0007936; otherwise, the minute sampled is in the interval [61, 360], and the estimated effort is the number counted divided by 0.0031746.

<table>
<thead>
<tr>
<th>Minute of the day</th>
<th>Anticipated relative fishing effort</th>
<th>Anticipated proportion of total effort</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>0.0007936</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>0.0007936</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>0.0007936</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>0.0007936</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>60</td>
<td>10</td>
<td>0.0032936</td>
</tr>
<tr>
<td>61</td>
<td>40</td>
<td>0.0031746</td>
</tr>
<tr>
<td>62</td>
<td>40</td>
<td>0.0031746</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>360</td>
<td>40</td>
<td>0.0031746</td>
</tr>
</tbody>
</table>

Total 12,600 1.0

1 Anticipated proportion of total effort = entry in column 1, row \( t \) per 12,600 (the total of the entries in column 2).
FIGURE 4 — Method of scheduling the survey agent’s starting location in a progressive count survey of angler effort in which the agent travels at location-specific speed. The bold line represents the travel path (s) necessary to arrive at a given location, s, if the survey agent starts in location 0 and travels in a specified direction at prespecified location-dependent speed. To pick a starting location, a random number u is picked from a uniform distribution, \( u \sim U(0, 1) \) (horizontal dotted line), and the starting location is taken to be \( s_0 = r^{-1}(u) \) (vertical dotted line).

Note that if uniform probabilities were used, each milestone would be assigned a sampling probability of 1/360 = 0.0027778. The Horvitz-Thompson estimate (equation 4) would be given in (angular-minutes) by

\[
\hat{f}_{s0} = \frac{\text{number counted}}{0.0027778},
\]

which is exactly the same is the usual estimate given by equation (1):

\[
\hat{f}_s = \text{number counted} \times 360.
\]

An additional example of the use of the Horvitz-Thompson estimator (for an access point survey) can be found in Pfeiffer (1967).

Discussion

Scheduling in the progressive count method — The first example illustrates that, for a fairly realistic situation, the bias caused by improper scheduling of the count can be appreciable (≤ 10%) when counting time is substantial. The bias would have been worse if angling effort had been low for a longer time at the beginning of the 2-h period or if the difference between highest and lowest effort had been larger. The bias would have been less if the time required to make a count were smaller. However, this bias is entirely avoidable by using proper survey design.

We suggest two methods for scheduling counts. The first method, picking one of a set of discrete starting times, requires that the starting location also be picked randomly. This method will often be referred to as the second (wrap-around) method. For example, if anglers are counted from an airplane in a way that the airplane would not want to interrupt the flight and thus would not want to use the second method.

Nonuniform travel speed when making progressive counts — Following Robson (1961) we described the progressive count method as requiring the survey agent to travel at a constant speed that is greater than the travel speed of the anglers. However, Robson (1961) pointed out that the progressive count approach can accommodate the more general situation in which the survey agent’s travel speed is a predetermined function of location in independent of the time of day, direction of travel, and the number of anglers present. The agent’s speed must be greater than the anglers’ travel speeds. For this more general case the agent’s starting location must be randomized in a particular way. Suppose the function \( r(s) \) indicates the survey agent’s travel time from the origin to location \( s \) (given the direction of travel). The inverse of this function, \( r^{-1}(u) \), indicates the location at which the survey agent would end up if the agent started at the origin and traveled for \( u \) units of time. The procedure for randomly picking the starting location is to pick a uniformly distributed random variable \( u_0 \sim U(0, 1) \), and then take the starting location \( s_0 = r^{-1}(u_0) \). For example, suppose the travel-time-location relationship is as shown in Figure 5. If a random number \( u_0 \) is selected from the interval (0, 1), then the corresponding location \( s_0 \) would be selected as the starting location.

To our knowledge, no one has yet designed a progressive count creel survey that accounts for prespecified nonuniform travel speed of the survey agent as a function of location. The approach has a great deal of appeal because external factors such as traffic regulations, obstructions, and detours may impose constraints on travel speeds. In addition, there may be certain areas where the habitat is known to be of a special quality and one may want to adjust the agent’s travel speed in relationship to the anticipated fishing effort in these special areas.

Variance — We deferred the question of estimating the variance of the estimated fishing effort...
until now in order to concentrate on the logic of the survey methods. The variance depends, of course, on the survey design. In general, one wants to estimate the total fishing effort over some large block of time (such as a month or a season) by sampling portions of the fishery in each of several days. A common procedure is to use two-stage sampling in which the primary sampling unit is the day and the secondary sampling unit is the moment when the count is made (or the moment of location combination where the count is begun). There are two components to the variance in this case: the variance of the within-day estimates and the between-day variance. If several independent instantaneous counts are made within each of several of the days sampled, then it is possible to estimate the components of the variance and the total variance (see Cochran 1977 for a discussion of two-stage sampling). However, when a single count is made for each primary sampling unit (i.e., day) sampled by the instantaneous count method, it is not possible to obtain unbiased estimates of the individual components. One can compute the sample variance of all the estimated daily efforts and use this to obtain a conservative (positively biased) estimate of the variance of the total estimated effort. These results correspond to two-stage (cluster) sampling when there is one observation at the second stage (see Cochran 1977: 278). The same situation occurs if a progressive count sampling scheme is used instead of the instantaneous count; one cannot estimate the components of variance if one makes only a single count per day sampled, but one can make a conservative estimate of the total variance. More formally, let the estimate of the total effort over the entire season be given by

\[ \hat{\text{total}} = N \bar{\tau} \]

where \(N\) is the number of days in the season and \(\bar{\tau}\) is the arithmetic mean of the daily estimates of fishing effort. Then a conservative estimate of the variance of the estimate of total effort is given by

\[ \hat{\text{V}}(\text{total}) = \frac{N^2 \hat{\sigma}^2}{\bar{\tau}^2} \]

where \(\hat{\sigma}^2\) is the estimated variance of the daily estimates of effort:

\[ \hat{\sigma}^2 = \frac{\sum (\bar{\tau}_i - \bar{\tau})^2}{n - 1} \]

Here, \(\bar{\tau}_i\) is the estimate of the fishing effort in the \(i\)th day sampled. Note that it is not appropriate to use a finite population correction in the variance estimator in this situation.

Nonuniform sampling probabilities for instantaneous counts. — It is a common procedure to pick primary sampling units, e.g., days, with nonuniform probabilities (see Malvessos et al. 1978). Here, we pointed out that it is also possible to pick the secondary sampling units (e.g., instants of the day) with nonuniform probabilities. To our knowledge, this has never been done although one might suspect that the pattern of fishing effort over the course of a day would be better known than the pattern over the course of a week.

We do not necessarily recommend use of nonuniform sampling probabilities at the first or second stage. It is true that, when the sampling probabilities are closely related to the amount of fishing effort in the sampling units, this approach can be very efficient. But, as Cochran (1977) and others have pointed out, nonuniform probability sampling (ppx sampling) can also lead to estimates with huge variances if inappropriate sampling probabilities are used. Also, it is worth noting that many crew surveys have multiple objectives. If nonuniform probabilities are used when selecting sampling units, then this design must be explicitly accounted for in the estimation procedure for all quantifies of interest. The sampling probabilities used might provide for very efficient estimation of one parameter but produce large variances for other parameter estimates. For example, one might wish to estimate fishing effort from charter boats and from private boats separately, and these types of boats may have different patterns of activity over time.

For completeness, we mentioned that it is possible to use the Hertzel-Thompson estimator for the modified progressive count method. A block of time is selected with nonuniform probability from among the \(n\) possible starting times, and the starting location on the travel circuit is selected with uniform probability. However, it is not clear how often this would provide an efficient estimator.

Final comments. — In reviewing numerous reports on crew surveys, we have often discovered statistical flaws in the design and analysis. Most troubling, however, was the most reports supplied so few details of the statistical procedures that it was impossible to determine exactly what was done. We feel strongly that the standards of documentation must be improved.
Acknowledgments

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References


Appendix 1: The Instantaneous Count Method

In this appendix we prove that the instantaneous count method provides unbiased estimates of fishing effort.

Consider that N anglers fish some time during the day, angler i fishes at location x (0 < x < E) between times t_{0i} and t_{1i} (0 < t_{0i} < t_{1i} < T). Note that the anglers are assumed to be stationary. The duration of fishing activity for angler i is \( t_{1i} - t_{0i} \). These angler trips can be represented geometrically as in Figure A1.1. Now, suppose an instance of time, t, is randomly selected from the interval (0, T), with uniform probability density, \( f(t) = 1/T \), and the number of anglers fishing is counted as that instant of time, resulting in a count \( n(t) \).

![Figure A1.1: Representation of time and space.](image)

Figure A1.1. Representation of time and space. If an instantaneous count is made at time t the result will be no anglers counted. Note that an angler trip represented by a horizontal line implies that the angler is stationary.

Leib 's' is kept as 1 if the ithangler is counted, then the expected value of \( h_i \) is, by definition,

\[
E(h_i) = \text{Prob}(h_i = 1) \times 1 + \text{Prob}(h_i = 1) \times 0 = \text{Prob}(h_i = 1),
\]

where Prob is probability. Furthermore, by definition,

\[
\text{Prob}(h_i = 1) = \int_0^T \frac{1}{T} dt - \int_0^{t_{0i}} \frac{1}{T} dt + \int_{t_{1i}}^{t_{0i}} \frac{1}{T} dt + \int_{t_{1i}}^T \frac{1}{T} dt
\]
estimating sportfishing effort

$$E(C, T) = TE(C) + \sum_{j=1}^{N} \sum_{i=1}^{K} E(b_i)$$

We note that this proof is also valid if the anglers move while fishing. This is because, at the instant the count is made, all anglers are counted, so it does not matter where the anglers are located.

Appendix 2: The Progressive Count Method

Here, we prove that the progressive count method

with constant agent travel speed provides an unbiased estimator of the fishing effort that occurs during the time the count is being made. Our proof follows closely the geometric arguments presented by Robson (1961). We begin with the case in which anglers are stationary.

Consider the angler population to be as in Appendix 1; however, rather than making an instantaneous count (i.e., determining all anglers present at a precise instant), the survey agent makes a progressive count by starting at a random location, \( s_0 \), and following a route such that all places where anglers can fish are viewed exactly once. The agent travels at constant speed starting at time 0 and ending at time \( T \). Two such possible routes are shown in Figure A2.1. One where the starting location, \( s_0 \), happens to be 0, and the other where \( s_0 \) happens to be \( a \). Note that the set of all possible trajectories through space and time is a set of parallel lines by virtue of the fact that travel speed is constant and that the entire length of the fishery, \( S \), must be visited.

We start by showing that if the anglers are stationary and a progressive count is made as described above with a random starting location chosen from a uniform distribution, \( s_0 \sim U(0, S) \), then the probability that the \( j \)th angler is counted, \( E(b_j) \), is equal to \( L_0/T \). This means that the probability an angler is counted is equal to the fraction of the day fished by the angler.

Case 1

In case 1 (see Figure A2.2), the line representing the fishing trip is located above the diagonal line extending from the point \((0, 0)\) to the point \((S, T)\).

$$E(b_j) = \int_{s_0}^{S} h_j \int_{0}^{T} \frac{1}{S} ds \, dt \quad \text{(by definition)}$$

Figure A2.1—Representation of the same angler population as in Figure A1.1. Two possible routes of the survey agent are shown. The solid line depicts the survey agent beginning at time 0 at location 0 and traveling at constant speed such that the agent follows a complete tour of the fishery at time \( T \). The dashed line depicts (in two parts) the survey agent beginning at time 0 at location \( a \), traveling at constant speed, and completing a complete tour of the fishery (i.e., returning to point \( a \)) at time \( T \). Note that the route is laid out as a "loop" or "circuit," such that location \( S \) is the same as location 0.
Therefore,

\[ t_1 = \frac{S}{T} t_1 + \frac{S}{T} t_2 \]

and

\[ t_2 = \frac{S}{T} t_2 + \frac{S}{T} t_1 \]

which imply, respectively,

\[ S_2 = S_1 - \frac{S}{T} t_1 \]

and

\[ S_1 = S_1 - \frac{S}{T} t_2 \]

By substitution,

\[ \frac{S_2 - S_1}{S} = \frac{\left( S_1 - \frac{S}{T} t_1 - S_1 + \frac{S}{T} t_2 \right)}{S} = \frac{t_2 - t_1}{T} = \frac{L_1}{T} \]

Case 2

In case 2 (see Figure A2.3), the line representing the fishing trip crosses the diagonal extending from the point (0, 0) to the point (S, T). Therefore, by substitution,

\[ S_1 = S_1 - \frac{S}{T} t_1 = \frac{L_1}{T} \]
of a single angle trip time $t_1$ and ending at the angle's trip with the angle at location $S_i$ and the angle's count the trip is between 0 and $S_i$.

$S = \int_{t_1}^{t_2} b \frac{1}{S} dt$

Case 1

In case 3, the line representing the fishing trip is entirely below the diagonal line extending from the point $(0, 0)$ to the point $(S, T)$. This case is the same as case 1.

By the same logic used in Appendix 1, it follows that the expected value of the estimator of fishing effort, $C_T$, is

$E(C_T) = TE(\sum_{i=1}^{N} b_i) = T \sum_{i=1}^{N} b_i \frac{1}{S} = \sum_{i=1}^{N} \frac{b_i}{S}$

which is the total effort.

Note that, although we have assumed that angles are stationary, the results hold for the more general situation in which angles may move about during the day, provided that they do not fish while moving. This is because the method is not concerned with the number of fishing trips but only the number of hours of fishing effort. Thus, an angle who fishes at one location for a while and then moves to another location can be treated as equivalent to two separate angles.

We may consider that angles may move about while in the process of fishing (e.g., when trolling). There are two cases to consider. In the first case, the angle's rate of travel while fishing is less than that of the crew survey agent. In the second case, the angle travels faster than the survey agent.

We will show that the progressive count estimator $C_T$ is an unbiased estimator of the fishing effort when the angles travel at speeds less than the constant speed of the survey agent and the agent's starting location and direction of travel are chosen randomly.

As before, it suffices to show that the expected value of $b_i$ is $L_i / T$. Consider the angle trip depicted in Figure A2.4A, for which the starting location of the angle is $a$ and the ending location is $S$.

Suppose the flip of a coin is used to determine with probability equal to one half the direction of travel. Then $a$ will move in the direction $0 \rightarrow a \rightarrow b \rightarrow S$. Then the expected value of $b_i$ given the direction of the agent's travel is as given before:

$E(b) = \int_{t_1}^{t_2} \frac{1}{S} b_i \, dt = \int_{0}^{t_1} \frac{1}{S} b_i \, dt + \int_{t_1}^{t_2} \frac{1}{S} b_i \, dt$

$= \frac{s_2 - s_1}{S}$.

Also, as before,

$s_1 = \frac{T}{2} + s_2$, so,

$a = S \frac{t_1}{t_2} + s_2$ and $s_2 = a - \frac{S}{2} (t_2 - t_1)$. 
\[
\begin{align*}
    b &= S \frac{t_2}{T} + s_1 \quad \text{and} \quad s_1 = b - S \frac{t_1}{T}, \\
    a &= S \frac{t_1}{T} + s_2 \quad \text{and} \quad s_2 = a - S \frac{t_2}{T}.
\end{align*}
\]

Therefore,
\[
\begin{align*}
    E(h) &= \frac{a - S \frac{t_1}{T} - b + S \frac{t_2}{T}}{S} \\
    &= \frac{a - b}{S} + \frac{t_2 - t_1}{T}.
\end{align*}
\]

On the other hand, if the survey agent travels in the opposite direction, \(S = b \to a \to T\) (see Figures A2.4B), then the expected value of \(h\) is
\[
\begin{align*}
    E(h) &= \int_0^{\frac{T}{2}} h \frac{1}{S} dt + \int_{\frac{T}{2}}^{T} h \frac{1}{S} dt \\
    &= \int_0^{\frac{T}{2}} \frac{1}{S} dt + \int_{\frac{T}{2}}^{T} \frac{1}{S} dt \\
    &= \frac{t_2 - t_1}{S}.
\end{align*}
\]

as before. Also,
\[
    s_1 = s_0 - \frac{S}{T};
\]

so,
\[
\begin{align*}
    a &= s_1 - S \frac{t_1}{T}, \\
    b &= s_2 + S \frac{t_2}{T}.
\end{align*}
\]

Therefore,
\[
\begin{align*}
    E(h) &= \frac{-a - S \frac{t_1}{T} + b + S \frac{t_2}{T}}{S} \\
    &= \frac{b - a}{S} + \frac{t_2 - t_1}{T}.
\end{align*}
\]

and the expected value of \(h\) averaged over the two possible directions of travel is
\[
\begin{align*}
    E(h) &= \frac{(a - b)}{2} + \frac{t_2 - t_1}{T} \\
    &= \frac{(b - a)}{2} + \frac{t_2 - t_1}{T} \\
    &= \frac{t_2 - t_1}{T} = \frac{L_u}{T};
\end{align*}
\]

as required. To complete the proof, one can use the same arguments to verify that the expected value of \(t_1\) is \(L_u/T\) for various possible starting and ending locations (i.e., crossing the diagonals or not).

The same approach can be used to establish that the progressive sound method provides biased estimates when the anglers in the process of fishing move faster than the survey agent.