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ESTIMATING TOTAL MORTALITY FROM LENGTH DATA WHEN SPAWNING OCCURS ANNUALLY

by

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Methods for estimating the total instantaneous mortality rate, Z, from length-frequency data have been available since Beverton and Holt (1956) derived the formula

$$Z = \frac{K(Linf - Lm)}{Lm - Lc} \qquad (1)$$

where K and Linf are parameters of the von Bertalanffy growth curve, Lm is the mean length of fish above the length Lc, and Lc is a "knife-edge" recruitment size. This and other approaches, which assume continuous recruitment (i.e. at all times of the year), were reviewed by Pauly (1983) and Hoenig et al (1983).

When recruitment occurs annually (once a year), an analogous discrete model can be derived by equating the mean length with a function of the growth parameters and mortality rate. Thus,

$$Lm = \left(\sum_{i=t_c}^{t_u} N_i Lm_i\right) / \left(\sum_{i=t_c}^{t_u} N_i\right)$$

$$= \left(\sum_{i=t_c}^{t_u} e^{-Zi} Lm_i\right) / \left(\sum_{i=t_c}^{t_u} e^{-Zi}\right)$$

$$= \frac{\left(\sum e^{-Zi} Lm_i\right)\left(1 - e^{-Z}\right)}{e^{-Zt_c} - e^{-Z(t_u+1)}} \qquad (2)$$

where Lm_i is the mean length at age i, N_i is the number of animals at age i, and t_c and t_u are the youngest and oldest ages fully represented in the sample. Note that Lm, the sample mean, is the mean of those fish whose ages are fully represented in the sample. This implies that one should choose a left truncation point (Lc) that lies in a "valley" between two peaks in a length-frequency distribution. Equation (2) can be solved iteratively using the solution of (1) as a preliminary estimate.

If von Bertalanffy growth equations are substituted for the Lm_i in (2), the formula becomes

$$Lm = Linf - \frac{Linf(1-e^{-Z})(e^{-t_c(Z+K)} - e^{-(t_u+1)(Z+K)})e^{Kt_o}}{(1-e^{-(Z+K)})(e^{-Zt_c} - e^{-Z(t_u+1)})}$$

(Note that an alternative growth model incorporating seasonal oscillations in growth (Pauly and Gaschütz 1979) can also be incorporated into (2). Finally, if there is no reason to believe the older age groups are under-represented, then t_u can be taken to be infinite and

$$Lm = Linf - \frac{Linf(1-e^{-Z})e^{-K(t_c-t_o)}}{1 - e^{-(Z+K)}}$$

Rearranging this to eliminate t_o gives

$$\frac{Linf - Lc}{Linf - Lm} = \frac{1 - e^{-(Z+K)}}{1 - e^{-Z}} \qquad (3)$$

Equation (3) can be solved iteratively using (1) to determine a preliminary estimate.

Using equation (1) as an approximation to (3) results in a positive bias whose severity increases as Lc approaches Lm (Table 1).

Table 1. Effect of using continuous spawning-based estimator when reproduction occurs annually. K = 0.3 yr^{-1}, Linf = 40 cm; t_o = 0. Table gives estimates of Z derived from equations (1) and (3).

	Lc = 10 cm		15 cm		20 cm	
Lm	eq. (1)	eq. (3)	eq. (1)	eq. (3)	eq. (1)	eq. (3)
15.0	1.5	.83	–	–	–	–
20.0	.60	.42	1.20	.71	–	–
25.0	.30	.23	.45	.33	.90	.57
30.0	.15	.12	.20	.16	.30	.23

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