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Utility-per-Recruit Modeling: A Neglected Concept

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Abstract.—Yield-per-recruit analysis consists of summing or integrating the relative weight of the catch over all ages exploited in a fishery. Utility-per-recruit analysis is a generalization of this in which the utility or value (not necessarily monetary) of the catch, instead of the weight, is accumulated over age. The utility function of age can take many forms. For a sport fishery, age-specific ratings of fish quality can be derived from angler interviews in order to construct the utility function. In commercial fisheries, utility would most generally be taken to be the net income per recruit (revenues minus costs). However, it will often be of interest to examine per-recruit revenue alone because this is easy to obtain. In several important cases, the computations can be performed analytically.

The yield-per-recruit analysis developed by Beverton and Holt (1957) is an important tool of fisheries managers because it relates output from a fishery to control variables such as fishing mortality and age (size) at recruitment. The classic analysis assumes that only the total weight of the harvest is important. That is, the value per gram of fish is independent of the age or size of the individual fish. This may be unreasonable. In sport fisheries, anglers are likely to have a distinct preference for larger fish so that an analysis that considers only the total weight harvested, and not the size structure of the catch, is likely to have little relevance. Similarly, whereas total yield in weight may be of interest in a fishery producing fish meal, it is an inadequate criterion for many fisheries where consumers have distinct size preferences that are reflected in the market prices of the fish.

A useful generalization would be to consider the utility obtained per recruit. That is, the yield obtained at each age should be weighted by an appropriate measure of value (not necessarily monetary). Indeed, Beverton and Holt (1957) incorporated market values (revenues) and costs into their analytical model to determine profits from the North Sea fisheries for plaice *Pleuronectes platessa* and haddock *Melanogrammus aeglefinus*. Jensen (1981) used a dynamic pool model in which trophy-sized fish in a sport fishery were assigned especially high values. Botsford and Hobbs (1986) maximized utility per recruit (defined as revenues minus costs) subject to a constraint on egg production. Various authors (e.g., Beverton and Holt 1957; Fox 1972) have incorporated the relationship between dressed weight and total weight into yield-per-recruit calculations to model more satisfactorily the actual utility derived from the resource. However, to our knowledge, there is no general and systematic treatment of utility-per-recruit modeling available in the fisheries literature.

In this paper, we develop the mathematical formulation for a general utility-per-recruit model. The computational procedure may involve performing a numerical integration. However, for many of the likely applications, solutions can be obtained either analytically or by making use of tables of the incomplete beta function. We then consider examples involving a commercial fishery for yellowtail snapper *Ocyurus chrysurus* and a sport fishery for sauger *Stizostedion canadense*. We show that conclusions drawn from utility-per-recruit analysis can be substantially different from those derived from classic yield-per-recruit analysis.

Utility-per-Recruit Models

The classic equilibrium yield-per-recruit model of Beverton and Holt (1957) can be expressed as

$$Y/R = e^{-M(t_c - t_r)} \int_{t_c}^{t_u} F W_t A_t \, dt; \qquad (1)$$

Y/R is the realized, steady-state yield in weight per recruit; F and M are the constant instantaneous fishing and natural mortality rates (time^{-1}), respectively; W_t is the weight of an individual fish at age t; A_t is the relative abundance at age t; t_c is the age at which the animals become vulnerable to the fishing gear if there is knife-edged recruitment (i.e., if no animals are caught below the age t_c and animals older than t are fully vulnerable to the gear); t_u is the oldest age vulnerable to the fishing gear; and t_r is the age at recruitment (age when animals become potentially exploitable). The mortality rates can be made age-specific in this and all subsequent models considered, provided that appropriate data are available. The factor $e^{-M(t_c-t_r)}$ will be referred to as R' in this paper. In usual practice,

$$W_t = aL^3 = W_\infty[1 - e^{-K(t-t_0)}]^3$$

or

$$W_t = aL^b = W_\infty[1 - e^{-K(t-t_0)}]^b,$$

and

$$A_t = e^{-(F+M)(t-t_c)}, \qquad t > t_c;$$

W_∞ (asymptotic weight), K (growth coefficient), and t_0 are parameters of the von Bertalanffy growth equation, and a and b are coefficients in the allometric equation relating weight to length (Ricker 1975). Note that the constants F, W_∞, and $e^{(F+M)t_c}$ can be factored out of the integral.

We now consider models for the total (gross) utility or value of the catch per recruit. Later in this section, we discuss models for net utility. Computation of utility per recruit (U/R) involves substituting a utility-at-age function, U_t, for the weight at age in equation (1). Thus,

$$U/R = R'F \int_{t_c}^{t_u} U_t A_t \, dt. \tag{2}$$

Note that in the classic yield-per-recruit analysis, it is implicitly assumed that only the total weight of the catch is of importance and not the age (size) distribution of the catch.

The utility function U_t can be any function of age (and hence of size) selected by the modeler. Calculation of utility per recruit may require use of a numerical method of integration on a computer. This should not be difficult even on a microcomputer when a commercially available package such as IMSL (IMSL 1984) is used. Alternatively, utility per recruit can be calculated in a step-wise manner analogous to Ricker's (1975) yield-per-recruit model.

An interesting case occurs where the utility is considered proportional to a power function of the length,

$$U_t = a^*L^{b^*} = a^*L_\infty^{b^*}[1 - e^{-K(t-t_0)}]^{b^*},$$

L_∞ being the asymptotic length. If the exponent b^* is equal to that in the formulation for weight at age (3 or, more generally, b), the utility per recruit is directly proportional to the yield per recruit and nothing is gained by considering utility instead of yield. When $b^* \neq b$, the utility-per-recruit analysis will provide conclusions different from those of the yield-per-recruit approach. For example, one can specify that angler satisfaction (utility) per gram increases with the size of the animal, so that $b^* > b$. Note that, for this utility function, the calculation of utility per recruit can be accomplished in exactly the same manner as the yield-per-recruit computations, for example, by making use of the incomplete beta function as described by Jones (1957). Tables of the incomplete beta function were tabulated by Wilimovsky and Wicklund (1963).

An important class of models arises when a polynomial is used to approximate an arbitrarily complex utility function of age. Analytical solutions for this class of models can be obtained by integrating by parts. Thus, if the utility function is given by the kth-order polynomial

$$U_t = \sum_{i=0}^{k} \beta_i t^i,$$

where the β_i are regression coefficients, then

$$U/R = R'F \int_{t_c}^{t_u} \left[\sum_{i=0}^{k} \beta_i t^i\right] e^{-(F+M)(t-t_c)} \, dt$$

$$= R'F \sum_{i=0}^{k} i!\, \beta_i \sum_{j=0}^{i} \cdot \frac{(-1)^j[e^{-(F+M)(t_u-t_c)}t_u^{i-j} - t_c^{i-j}]}{(i-j)!\,(-F-M)^{j+1}}.$$

(Note that $0! = 1$.)

A particular case of a polynomial utility function would be one in which the utility is considered constant over some range of ages. For example, shrimp and some other shellfish are usually graded and marketed in discrete commercial size categories. The price per weight increases with the size of the animal in a step function. This utility (price) information can easily be incorporated into a utility-per-recruit analysis by breaking the computations into parts. Thus, total utility per recruit is equal to the sum of the utilities obtained from each range of ages. The utility obtained per recruit in the first range of ages (corresponding to the range of sizes in the first [smallest] market category) is given by

$$(U/R)_1 = R'U_1F \int_{t_c}^{t_1} e^{-(F+M)(t-t_c)} \, dt$$

$$= R'U_1F(F+M)^{-1}[1 - e^{-(F+M)(t_1-t_c)}]; \tag{3}$$

U_1 is the utility per shrimp for shrimp in the first size category. The utility obtained in the second range of ages proceeds along similar lines, except that the abundance of animals recruited into the age range must be computed on the basis of the survival through the previous range of ages:

$$(U/R)_2 = R'e^{-(F+M)(t_1-t_r)}U_2F\int_{t_1}^{t_2} e^{-(F+M)(t-t_1)}\,dt$$
$$= R'e^{-(F+M)(t_1-t_r)}U_2F(F+M)^{-1}$$
$$\cdot[1 - e^{-(F+M)(t_2-t_1)}]. \quad (4)$$

Another possible formulation for utility is the logistic model. As discussed below, this may be useful in the analysis of sport fisheries.

Angler Satisfaction in Sport Fisheries

It is difficult to measure the satisfaction derived by an angler from a fishing trip. Economists sometimes express satisfaction in economic terms by estimating the amount of money an angler would be willing to expend for a given type of fishing trip. However, it is not always clear that monetary values are appropriate for quantifying the intangibles of a fishing trip.

An alternative approach would be simply to express satisfaction in arbitrary units. This could be established by assuming a functional form for satisfaction related to a factor such as length (which, in turn, can be related to age). For example, the utility of an individual fish might be a power function of its length. This might be appropriate in a fishery in which trophy-sized fish are in high demand. Another possible formulation would be to express satisfaction as a logistic function of length or age. Thus,

$$U_t = \frac{1}{1 + e^{-Q(t-t^*)}}; \quad (5)$$

U_t is the mean attitude toward (utility of) fish of age t, t^* is the age at which the inflection occurs, and Q is the rate coefficient. This is appealing because parameter estimation is readily accomplished by asking anglers to rate, say on a scale of 1 to 5, the satisfaction derived from a fish of a particular size. The distribution of mean satisfaction scores tends to resemble an S-shaped function of length and of age (Donald Pereira, Minnesota Department of Natural Resources, personal communication). Thus, this information rests on an assumption that there is a limit to the satisfaction derived from a given type of fish. It would be appropriate for a "panfish" fishery in which attention is not focused on trophy sizes.

The utility for a fishing trip could be assumed to be equal to the sum of the utilities derived from the individual fish caught. That is, the contentment of an individual angler is modeled by

$$U_i = \sum_j U_{ij};$$

U_i is the total utility derived by angler i and U_{ij} is the utility derived by angler i from fish j.

This is a reasonable model for individual satisfaction that accounts explicitly for both size and quantity of the catch. It is also appealing because the parameters can be estimated easily. More complicated models (involving, for example, interaction between fish size, quantity of fish caught, and number of trips) may be more realistic, but it would be more difficult to estimate the parameters and more difficult to construct the models. The situation also becomes more complicated when bag limits are imposed.

To construct a utility-per-recruit model, it is necessary to average the fish-age-specific utilities over the angler population. The resulting function, U_t, is then incorporated in equation (2) as before.

Net Utility Models

In many instances, interest will be centered on the net utility obtained per recruit, defined as the revenue minus the cost. In a simple model, cost might be proportional to effort which, in turn, is proportional to fishing mortality. Then utility per recruit would be computed as

$$U/R = (R'\int U_tFA_t\,dt) - (cf/R)$$
$$= R'\int U_tFA_t\,dt - c(F/q)/R;$$

c is a constant relating cost to fishing effort (f), q is a constant of proportionality relating fishing mortality to effort, and R is the magnitude of recruitment. More generally, cost can be broken into a fixed component (C) and a component proportional to fishing effort. Thus,

$$U/R = R'\int U_tFA_t\,dt - (C + cF/q)R.$$

Other models are possible; the above are intended only to suggest some useful, simple cases.

Note that to convert total costs to a per-recruit basis, it is necessary to determine the magnitude of the average recruitment. For this reason, one might often begin an analysis by computing gross utility per recruit. When information on average recruitment becomes available, one can convert to net utility by subtracting the per-recruit costs.

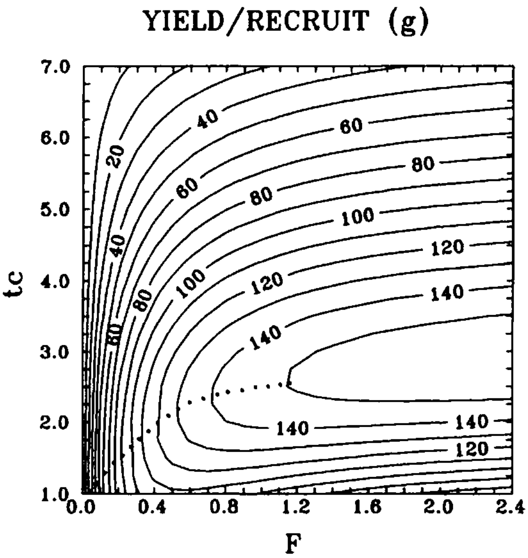

FIGURE 1.—Yield-per-recruit isopleths for yellowtail snapper. Isopleth locations were approximated based on the statistical contouring procedure of kriging; F is instantaneous fishing mortality; t_c is age at vulnerability to fishing gear (years). Dotted line indicates eumetric fishing (t_c chosen to maximize yield for a given F). The highest computed yield is 157 g, which occurs at F = 2.4, t_c = 3.0 years.

TABLE 1.—Price data for snapper fillets (mixed species) available in the Boston, Massachusetts, wholesale market in 1987. Columns 1 and 2 correspond to the original data (NMFS 1987) and the other columns to the estimated, age-specific, utility values.

Weight of fillets (ounces)	Price/pound (US$)	Whole weight of fish (g)[a]	Price/fish (US$)[b]	Age (years)[c]
2	2.30	381	0.58	2.56
3	2.55	497	0.96	2.91
4	2.75	613	1.38	3.23
5	3.05	730	1.91	3.54
6	3.45	846	2.59	3.84
7	3.70	962	3.24	4.13
8	3.90	1,078	3.90	4.42
9	3.90	1,194	4.39	4.71
10	3.90	1,310	4.88	5.01
11	3.60	1,426	4.95	5.30
12	3.60	1,542	5.40	5.61
13	3.10	1,658	5.04	5.91
14	3.10	1,775	5.42	6.24
15	3.10	1,891	5.81	6.58
16	3.10	2,007	6.20	6.92

[a] Whole weight (g) = 149 + [116 × fillet weight (ounces)] based on a regression fitted to data from E. Gaw (Merrit Seafood, Incorporated, personal communication): two fillets represent 34% of the whole weight of a 1-pound fish, 40% of a 2-pound fish, and 45% of a 4-pound fish.

[b] Price/fish = price/pound × 454 g/pound × whole weight (g).

[c] Age = $-\log_e[1 - (W_t/W_\infty)^{1/3}]/K = -\log_e[1 - (W_t/3{,}600)^{1/3}]/0.25$ based on the von Bertalanffy growth formulation and the parameter estimates in Munro (1983); W_t is weight at age t, W_∞ is asymptotic weight, and K is a growth coefficient.

Applications

In this section, we consider two examples that illustrate the generality of the utility-per-recruit concept and the diversity of possible formulations for utility as a function of age.

Price Structure and the Management of Jamaican Yellowtail Snapper

Munro (1983) studied the fishery biology of yellowtail snapper in Jamaica and estimated that the natural mortality rate, M, is 0.6·year^{-1}; the relationship between weight in grams and age in years is

$$W_t = 3{,}600(1 - e^{-0.25t})^3.$$

We calculated yield-per-recruit isopleths for this stock under the assumption that the age at recruitment (t_r) is equal to 1.0 year (Figure 1).

The Boston, Massachusetts, market value of snapper fillets (mixed species) depends on the size of the fish; animals of "dinner plate" size fetch the highest prices (Table 1, after NMFS 1987). For illustrative purposes, we assumed this price structure was an appropriate measure of utility for the yellowtail snapper fishery in Jamaica. Price per weight was considered a continuous function, and the weight of two fillets was assumed to account for 34–45% of the whole weight of the fish (Edward Gaw, Merrit Seafood, Incorporated, personal communication); thus the following polynomial regression was developed from the data in Table 1 to describe utility in dollars (U_t) as a function of age (t) in years:

$$U_t = 35.8 - 36.4t + 13.1t^2 - 1.90t^3 + 0.098t^4; \qquad 2.5 \le t \le 7$$

(Figure 2). This utility function and the above biological parameter estimates were used to construct utility-per-recruit isopleths (Figure 3).

If fishing mortality is fixed in the range $0.6 < F < 1.2$, the best value of t_c based on yield calculations would be approximately 2.5 years, whereas the best t_c based on the utility criterion would be about 3.5 years. These values of t_c correspond to weights of 361 g and 714 g, respectively. If a manager chose t_c to be 2.5 years (based on yield), the utility obtained per recruit would be about 0.40. If the manager chose 3.5 years for t_c, the utility per recruit would be greater than 0.45. Use of the yield criterion to manage the fishery

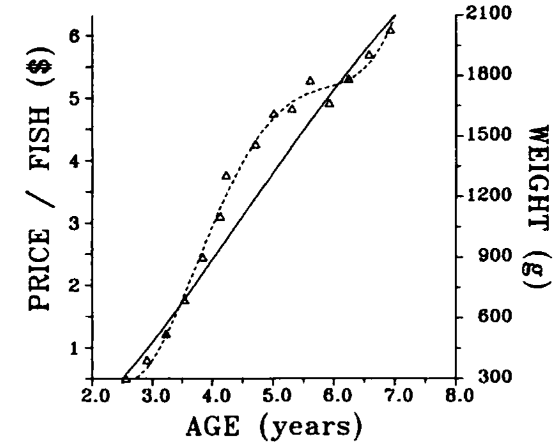

FIGURE 2.—Weight (solid line) and value (price, dashed line) of yellowtail snapper as a function of age (based on data in Table 1). Fitted line is a fourth-degree polynomial regression.

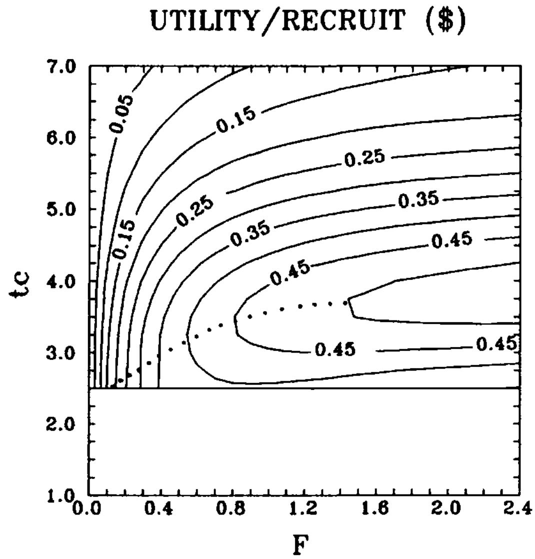

FIGURE 3.—Utility-per-recruit isopleths for yellowtail snapper. Isopleths were computed over the range of ages for which utility data were available. Isopleth locations were approximated based on the statistical contouring procedure of kriging; F is instantaneous fishing mortality; t_c is age at vulnerability for fishing gear (years). Dotted line indicates conditions analogous to eumetric fishing, i.e., fishing to maximize utility for a given value of F. The highest computed value is \$0.52, which occurs at $F = 2.4$, $t_c = 3.75$ years.

when utility is the appropriate criterion would result in a loss of utility in excess of 11%.

Angler Attitudes and the Management of a Sport Fishery for Sauger

Consider the sauger sport fishery at Lake of the Woods, Minnesota. Estimates of the von Bertalanffy growth parameters are $L_\infty = 48.8$ cm, $K = 0.167 \cdot \text{year}^{-1}$, and $t_0 = -0.43$ year. Average water temperatures are approximately 7.9°C, hence the natural mortality rate M estimated by the method of Pauly (1980) is $0.27 \cdot \text{year}^{-1}$. Estimated coefficients of the allometric equation are $a = 7.79 \times 10^{-3}$ and $b = 3.0$. It is believed that the current fishing mortality F is 0.53 year^{-1} and that the minimum acceptable size (L_c) is roughly 25–30 cm, corresponding to an age of first capture of 4–5 years (D. Schupp, Minnesota Department of Natural Resources, personal communication). The minimum size was determined from a sample of the creel; there is no legal restriction on minimum size. Yield-per-recruit isopleths based on this biological information suggest that a yield per recruit of 64.5–65 g would be achieved if F is 0.53 and the length of first capture is 25–30 cm (age 4–5 years). A value for t_c of 4.5 years would result in eumetric fishing (in which t_c is chosen to maximize yield for a fixed F) when F is 0.53 but would hardly change the yield (Figure 4). The best t_c for large F would be approximately 5.5 years.

The size-specific utility of saugers was measured by asking anglers to rate on a scale of 1 (low) to 5 (high) their satisfaction with their creeled fish. The scores were rescaled to the interval from 0 to 1

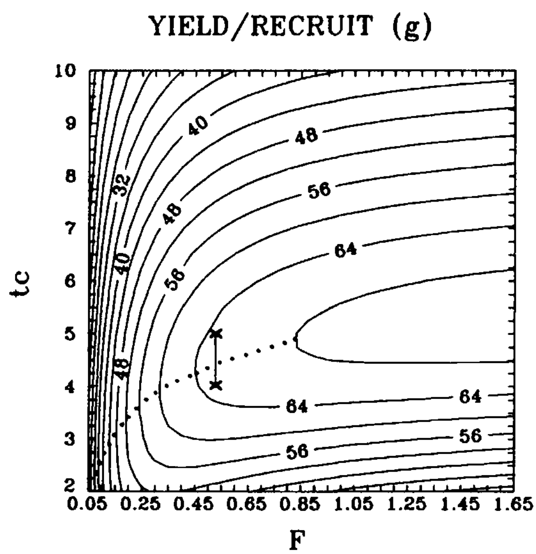

FIGURE 4.—Yield-per-recruit isopleths for sauger from Lake of the Woods, Minnesota. Isopleth locations were approximated based on the statistical contouring procedure of kriging; F is instantaneous fishing mortality; t_c is age at vulnerability to fishing gear (years). Dotted line indicates eumetric fishing (t_c chosen to maximize yield for a given F). Line connected by Xs represents estimated current conditions in the fishery.

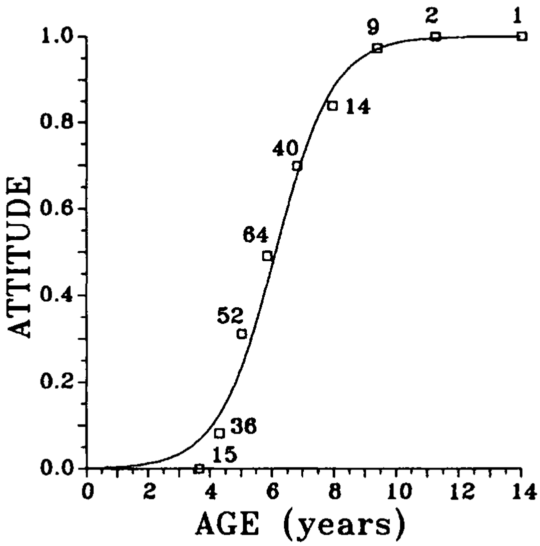

FIGURE 5.—Mean angler satisfaction rating (attitude) for sauger as a function of age of fish for the Lake of the Woods fishery. Satisfaction is expressed in arbitrary units (rescaled to the interval from 0 to 1) and was determined by asking anglers to rate creeled fish on an integer scale from 1 to 5. Fitted curve is a logistic model. Numbers refer to the number of responses for each age-class.

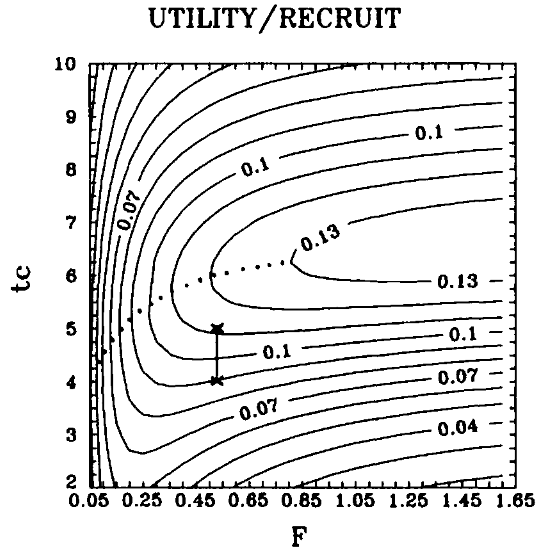

FIGURE 6.—Utility-per-recruit isopleths for sauger from Lake of the Woods, Minnesota. Isopleth locations were approximated based on the statistical contouring procedure of kriging; F is instantaneous fishing mortality; t_c is age at vulnerability to fishing gear (years). Dotted line indicates conditions analogous to eumetric fishing, i.e., fishing to maximize utility for a given value of F. Line connected by Xs represents estimated current conditions in the fishery.

and expressed as a function of age (determined by inverting the von Bertalanffy growth equation) (Figure 5). A logistic model (equation 5) was fitted to these data by linear regression after the logit transformation was applied to the U_t values. Parameter estimates were $t^* = 6.13$ years and $Q = 1.07 \cdot \text{year}^{-1}$. This utility function was used with the above estimates to compute utility-per-recruit isopleths (Figure 6). At $F = 0.53 \cdot \text{year}^{-1}$ and $t_c = 4$ years, the utility per recruit was just under 0.09 satisfaction units. A t_c value of 5 years would increase utility per recruit to a little over 0.11 satisfaction units. A t_c value of 6 years would result in a little over 0.12 satisfaction units per recruit and a condition analogous to eumetric fishing (i.e., t_c is chosen to maximize utility for a fixed F). A t_c value of 6 years corresponds to a minimum size of 32 cm. Thus, the current minimum size (25–30 cm, 4–5 years) accepted by the angling community results in a utility of 0.09–0.11 satisfaction units per recruit, whereas an increase in the minimum size to 32 cm would increase utility to over 0.12 satisfaction units. Furthermore, because the eumetric fishing line is quite flat, a minimum size of 32 cm (t_c approximating 6 years) would be close to optimal for all fishing mortalities greater than about 0.65. Note that if one accepted the yield-per-recruit analysis, and told anglers they might as well keep smaller fish (as young as 4–5 years and as small as 25–30 cm) instead of encouraging them to exercise self-restraint, then the utility per recruit would fall between 0.09 and 0.11 (an 8–25% decline in utility).

In addition to estimating the average utility obtained per recruit, the fishery manager may wish to examine the frequency distribution of utility values in the catch. This is computed by integrating the numbers in the catch over the range of ages corresponding to the values of utility of interest. Thus, if the interval of interest extends from age t_1 to t_2, the catch per 1,000 recruits ($1{,}000\, C_{t_1,t_2}$) is given by

$$
\begin{aligned}
1{,}000 C_{t_1,t_2} &= 1{,}000 e^{-M(t_1-t_r)-F(t_1-t_c)} F \cdot \int_{t_1}^{t_2} e^{-(F+M)(t-t_1)}\, dt \\
&= 1{,}000 e^{Mt_r+Ft_c} [F/(F+M)] \cdot [e^{-(F+M)t_1} - e^{-(F+M)t_2}].
\end{aligned}
$$

The distribution of utility values in the catch of saugers was computed for three scenarios in which average utility per recruit was held constant at 0.12 (Figure 7). These scenarios were chosen to

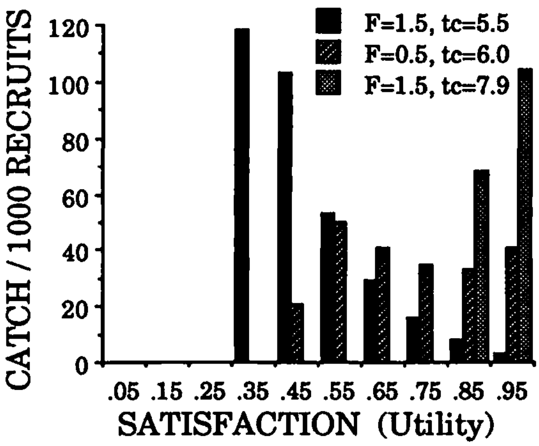

FIGURE 7.—Frequency distribution (per 1,000 recruits) of satisfaction (utility) values occurring in the catch of Lake of the Woods sauger for three scenarios. Average utility per recruit was held fixed at 0.12 satisfaction units. F is instantaneous fishing mortality; t_c is age at vulnerability to the fishing gear (years).

demonstrate that there may be many ways to achieve a given overall level of utility per recruit. A decision maker interested in providing a diversity of fishing opportunities may wish to maximize the utility per recruit while providing differing distributions of utility in different bodies of water.

Discussion

The examples presented in the previous section are somewhat simplistic. However, the barriers to improved analysis appear to be due largely to limitations in available data rather than to difficulties in theoretical development. It is instructive, therefore, to examine the limitations of the previous examples.

For the commercial fishery example, we considered revenues (prices) but ignored costs because cost data were unavailable. This model would be appropriate if costs were fixed because the calculated utility per recruit would be a linear function of the actual net utility per recruit. For example, if costs are largely determined by effort, and if effort is fixed, the utility-per-recruit analysis will identify the value of t_c that maximizes revenue and thus net utility.

In the same example, catchability was considered to be independent of age. However, for some fisheries, older animals may occupy a different habitat than younger ones and be exploited by different gear. In such cases, the fishing mortality rate and costs will be age-specific.

We also treated the age-specific prices of the fish harvested as constant rather than as a function of supply (quantity harvested). In the example considered, the Jamaican fishery was just one of several possible sources for the market. It was reasonable, therefore, to consider market price as being determined external to any particular fishery, as we did. Also, in at least some fisheries, the price elasticity to supply is low (Ioannides and Whitmarsh 1987), thus justifying our choice of model.

In most commercial fisheries, production passes through the control of several user groups. Typically, this includes at a minimum the fishermen, wholesalers, and retailers. Each group has its own costs and revenues and consequently derives a group-specific utility per recruit from the resource. No additional theory is necessary to compute utility separately by group and, by doing so, the manager gets a more detailed picture of the implications of a management policy.

In the sauger sport fishery example, we assumed that knife-edged recruitment could be achieved at age t_c by imposition of a minimum size limit. Of course, some of the undersized fish released will suffer hooking mortality; we ignored this. However, this source of mortality can be added to the analysis (Clark 1983; Waters and Huntsman 1986) if the hooking mortality is quantified (e.g., see Payer et al. 1987).

Angler attitudes about fish of different sizes were determined by asking anglers about fish in the creel. Presumably this led to some bias because only fish of value were retained. A better method might be to ask anglers about fish in the possession of the interviewer. Careful consideration should be given to the method of collecting attitude data.

In addition to examining effects of fishing mortality and minimum size, the sport fishery manager may wish to evaluate other management options, including slot limits and fishing season length and timing. Both of these factors can be studied in the context of utility-per-recruit analysis. Indeed, the former was studied by Jensen (1981) for a Wisconsin trout fishery, where utility was assumed to take only two possible (arbitrary) values depending on whether or not the fish was considered to be of trophy size. Obviously, other utility functions could be employed in the calculations. Moreau (1985) made some progress towards a yield-per-recruit analysis for a seasonal fishery by incorporating a seasonal growth curve in the yield model. He integrated yield over the entire age range, starting at age t_c. However, to model a seasonal fishery, it would be more appropriate to break the computations into parts as suggested in equations (3) and (4). Thus, one would calculate the available stock at age t_c, the harvest during the fishing season for the first age-group exploited, the survival of the remaining fish to the beginning of the next fishing season, the harvest from the second age-class exploited, etc. The calculations may be lengthy but are not difficult.

No effort was made in this paper to assess the effects of uncertainty in parameter estimates on management conclusions. The techniques developed for standard yield-per-recruit computations could also be used for utility-per-recruit analysis. These techniques include the delta method (Seber 1982) and the Monte Carlo method of Restrepo and Fox (1988, this issue).

Acknowledgments

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